STATS (2) – June 2005

(1) a) $P(X=2) = \frac{e^{-2.6} \times (2.6)^2}{2!} = 0.25104...$

b) i) $2.5 \times 5 = 13 \Rightarrow Y \sim Po(13)$

ii) $P(Y \geq 15) = 1 - P(Y \leq 14)$

$= 1 - 0.6751 = 0.3249$

(2) H_0: Time of day has no effect on result (INDEPENDENT)

H_1: Time of day influences result (NON-INDEPENDENT)

As there are 2 rows × 2 columns, we need YATES' CORRECTION ($\nu = 1$)

Observed

	Afternoon	Evening	
Win	30	24	54
Lose	18	28	46
	48	52	100

Expected

	Afternoon	Evening
Win	25.92	28.08
Lose	22.08	23.92

Yates' Correction $|O-E| - 0.5$

	A	E
W	3.58	3.58
L	3.58	3.58

X^2 $(|O-E|-0.5)^2/E$

	A	E
W	0.4945	0.4564
L	0.5805	0.5358

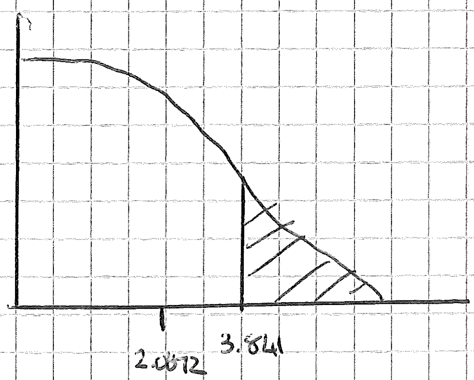
Calculated value of X^2 (Test Statistic) $= 2.0672$

Critical Value: $X^2_{(5\%)}(1) = 3.841$

$2.0672 < 3.841$

$\therefore$ Do not reject H_0

Not enough evidence at the 5% level to suggest the time of day has an effect on the outcome of a frame of snooker.



(3) a) From calculator:

$\Sigma x = 15.8$, $\Sigma x^2 = 25.0592$, $n = 10$

$\bar{x} = \frac{15.8}{10} = 1.58$ $s = 0.10282$

$s^2 = 0.010579$

b) $n = 10$, so $v = 9$

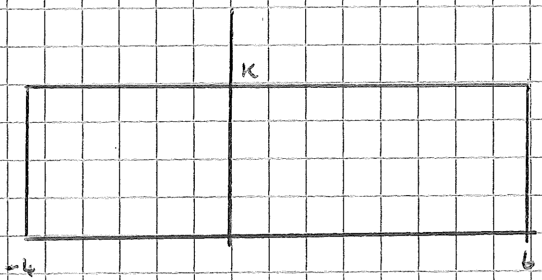
For 90% CI, look up 0.95, $t_9 = 1.833$

$CI = 1.58 \pm 1.833 \times \frac{0.10282}{\sqrt{10}}$

$= 1.58 \pm 0.0595...$

$= (1.52, 1.64)$

(4)



a) $k = 1/10$

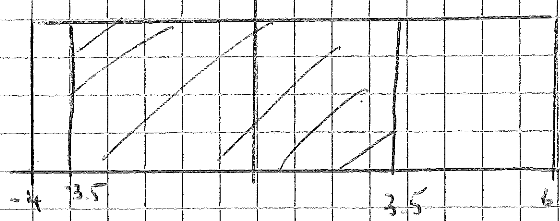
b) $E(X) = \frac{6 + (-4)}{2} = 1$

c) $P(X > 0) = 6 \times 0.1 = 0.6$

d) $P(|X| > 3.5)$

$= 1 - P(|x| < 3.5)$

$= 1 - 0.7 = 0.3$



(5) a) $E(R) = 1 \times (1/4) + 2 \times (1/2) + 4 \times (1/4) = 2.25$

$Var(R) = E(R^2) - [E(R)]^2$

$E(R^2) = 1 \times (1/4) + 2^2 \times (1/2) + 4^2 \times (1/4) = 6.25$

$\therefore Var(R) = 6.25 - 2.25^2 = 1.1875$

b) i)

x	1	1/4	1/16
$P(X=x)$	1/4	1/2	1/4

$X = 1/R^2$

$E(X) = 1 \times (1/4) + 1/4 \times (1/2) + 1/16 \times (1/4) = 25/64$

ii) $Area = 8/R\,(R + 8/R) = 8 + 64/R^2$

$E(Area) = 8 + 64\,E(1/R^2) = 8 + 64 \times 25/64 = 33$

(b) $H_0 : \mu = 568$

$H_1 : \mu < 568$ (1 tailed)

{t as don't know σ^2}

1% sig level, 1-tailed test, $\nu = 8 - 1 = 7$

• CRITICAL VALUE = -2.998

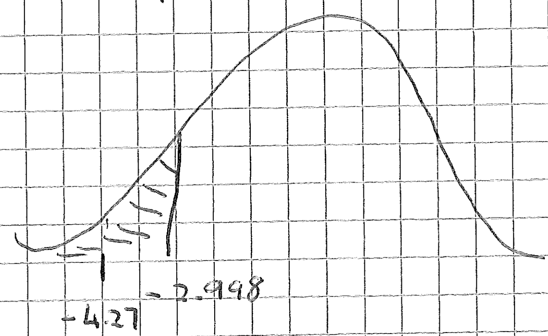
{$n = 8$
$\Sigma x = 4510$
$\Sigma x^2 = 254256.8$}

$\bar{x} = \frac{4510}{8} = 563.75$

$s^2 = 2.816 \rightarrow s^2 = 7.929$

• TEST STATISTIC =

$$t = \frac{563.75 - 568}{2.816/\sqrt{8}} = -4.27$$

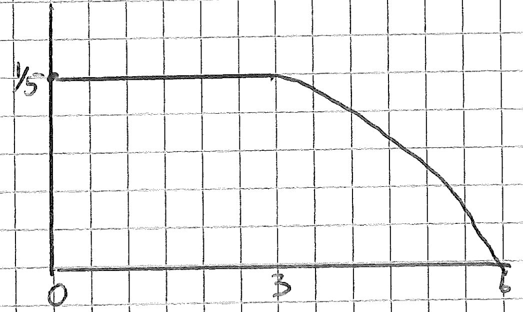


$-4.27 < -2.998$

∴ Reject H_0

There is evidence at the 1% significance level to suggest the average contents of the cartons have been reduced.

(7) a)



b) $P(T = 3) = 0$
as cumulative

c) $P(T \geq 3) = 1 - P(T < 3)$

$= 1 - (3 \times 1/5)$ From graph

$= 2/5$

d) As $P(T \leq 3) = 3/5$, median must be between 0 and 3

$$\int_0^m 1/5 \, dt = 0.5$$

$$\left[t/5 \right]_0^m = 0.5$$

$m/5 = 0.5 \Rightarrow m = 2.5$

e) Need Mean $= E(T) = \int_0^a x f(x)$

$$= \int_0^3 1/5 \, t \, dt + \int_3^6 1/45 \, t^2 (6 - t)$$

$= \int_0^3 \frac{1}{15}t \, dt + \int_3^6 \frac{2}{15}t^2 - \frac{1}{45}t^3$

$= \left[\frac{1}{10}t^2\right]_0^3 + \left[\frac{2}{45}t^3 - \frac{1}{180}t^4\right]_3^6$

$= \frac{1}{10}(9) + \frac{2}{45}(216) - \frac{1}{180}(1296) - \frac{2}{45}(27) + \frac{1}{180}(81)$

$= 2.55$

P(median < T < mean)

$= P(2.5 < T < 2.55)$

$= 0.05 \times 1/5$ (From graph)

$= 0.01$

(8) a) $H_0: \mu = 35$

$H_1: \mu \neq 35$ (2 tailed)

Z as we know $\sigma^2 = 12^2$

2 tailed test, 1% sig level, n = 100

CRITICAL VALUE $= \pm 2.5758$

$\bar{x} = 37.9$

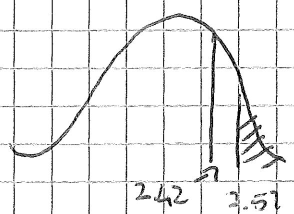
$\sigma^2 = 12^2$

$n = 100$

$H_0: \bar{X} \sim N(35, 144/100)$

TEST STATISTIC:

$z = \frac{37.9 - 35}{12/\sqrt{100}} = 2.42$



$2.42 < 2.575$

Do <u>not</u> reject H_0

Not enough evidence at 1% level to reject claim that average age is 35 years

b) Type II error means accept H_0 when it is false

So accept mean age is 35 years when it is not